Assignment 3

Hand in Ex 2.4, no. 8, Supp. Ex (3) by Feb 7.

- Exercise 2.3 no. 5, 7.

- Exercise 2.4 no. 1, 3, 8, 9.

Supplementary Exercises

1. Let

$$f^{(k)} \sim \frac{a_0^k}{2} + \sum_{n=1}^{\infty}(a_n^k \cos nx + b_n^k \sin nx) \ .$$

   Show that for an infinitely many times differentiable, 2π-periodic function f,

$$a_n^{2k} = (-1)^k n^{2k} a_n, \ k \geq 1, \quad a_n^{2k-1} = (-1)^{k+1} n^{2k-1} b_n, \ k \geq 0.$$

   Then find a similar formula relating b_n^k to a_n, b_n.

2. Verify that the following sequences are rapidly decreasing: (a) $x_n = 1/2^n$ and (b) $y_n = \sin n / n^n$.

3. Show that the cosine series
$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

   is equal to a sine series on $(0, \pi)$. Find this sine series and check if these two series are equal at 0 and π.